94 學年度 數學科教材教法
和角公式 - 942209005 天文所 廖家賢

1. $\sin(\frac{5\pi}{6}-\theta)+\sin(\frac{5\pi}{6}+\theta)=\cos\theta$

$$\sin(\frac{5\pi}{6}-\theta)+\sin(\frac{5\pi}{6}+\theta)$$
$$=\left(\sin(\frac{5\pi}{6})\cos(\theta)+\cos(\frac{5\pi}{6})\sin(\theta)\right)+\left(\sin(\frac{5\pi}{6})\cos(\theta)-\cos(\frac{5\pi}{6})\sin(\theta)\right)$$
$$=\left(\frac{1}{2}\cos(\theta)+\left(\frac{-\sqrt{3}}{2}\right)\sin(\theta)\right)+\left(\frac{1}{2}\cos(\theta)-\left(\frac{-\sqrt{3}}{2}\right)\sin(\theta)\right)$$
$$=\cos\theta$$

2. $\sin(67°)\cos(112°)-\cos(67°)\sin(112°)=?$

$$\cos(67°)\cos(112°)+\sin(67°)\sin(112°)$$
$$=\cos(67°-112°)$$
$$=\cos(-45°)$$
$$=\frac{1}{\sqrt{2}}$$

3. 已知$\triangle ABC$中，$\sin\angle A=\frac{4}{5}$且$\cos\angle B=\frac{12}{13}$，則$\sin\angle C=?$

因爲$\sin\angle A=\frac{4}{5}$，所以$\cos\angle A=\frac{3}{5}$。

因爲$\cos\angle B=\frac{12}{13}$，所以$\sin\angle B=\frac{5}{13}$。

$$\sin\angle C$$
$$=\sin(\pi-(\angle A+\angle B))$$
$$=\sin(\angle A+\angle B)$$
$$=\sin\angle A\cos\angle B+\cos\angle A\sin\angle B$$
$$=\frac{4}{5}\times\frac{12}{13}+\frac{3}{5}\times\frac{5}{13}$$
$$=\frac{48+15}{65}$$
$$=\frac{63}{65}$$

4. 利用和角公式求 $\cos 15° = ?$ 。

$$
\begin{aligned}
&\cos 15° \\
&= \cos(45° - 30°) \\
&= \cos 45° \cos 30° + \sin 45° \sin 30° \\
&= \frac{1}{\sqrt{2}} \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \frac{1}{2} \\
&= \frac{1+\sqrt{3}}{2\sqrt{2}} \\
&= \frac{\sqrt{2}\left(1+\sqrt{3}\right)}{4}
\end{aligned}
$$

5. 若 $\tan a = \frac{1}{3}$ $\tan b = -2$ ，則 $\tan(a+b) = ?$

$$
\begin{aligned}
&\tan(a+b) \\
&= \frac{\tan a + \tan b}{1 + \tan a \tan b} \\
&= \frac{\frac{1}{3} + (-2)}{1 + \frac{1}{3} \times -2} \\
&= -5
\end{aligned}
$$