8.? $|x| \geq 1$.· ´

$$2\text{arctan}x + \text{arcsin}\frac{2x}{1+x^2} = \pi\text{sgnx}.$$

Û . sgnx$x$j ! signum of x, Æ Y S R 7 :

$$\text{sgnx} = \begin{cases} -1 & ; \quad x < 0, \\ 0 & ; \quad x = 0, \\ 1 & ; \quad x > 0. \end{cases}$$

9.L'Hôpital Y \ g H c W g ´ :1

$$f(x) = x^2\text{sin}\frac{1}{x}, \qquad \phi(x) = x,$$

· ´

$$\text{lim}_{x\to 0}\frac{f(x)}{\phi(x)} = \lim_{x\to 0} x\text{sin}\frac{1}{x} = 0,$$

ß $\frac{f'(x)}{\phi'(x)}$Ï $x \to 0$« . e m .

$*$10.? $f(x)$° $g(x)$a l 7 $ Ú Õ :

(a)$f(x), f'(x), \ldots, f^{(n-1)}(x), g(x), g'(x), \ldots, g^{(n-1)}(x)$ a D $[a,b]$d U Ô .

(b)$f^{(n)}(x)$/ $g^{(n)}(x)$a D (a,b)d U D .

(c)$[a,b]$d s $n+1$µ * $x_0 < x_1 < \cdots < x_n$ ± $f(x_i) = g(x_i), i = 0, 1, 2, \ldots, n.$

· ´ (a,b)d r s ! * ξ± $f^{(n)}(\xi) = g^{(n)}(\xi)$.

Û . $n = 1$« ; p Rolle's theorem.

$*$11.? $f(x), g(x), h(x)$a p $[a,b]$9 z U Ô É ¨ ,# a D (a,b)9 K . ¢ . · ß (a,b) d r s ! ¨ ξ±

$$\begin{vmatrix} f'(\xi) & g'(\xi) & h'(\xi) \\ f(a) & g(a) & h(a) \\ f(b) & g(b) & h(b) \end{vmatrix} = 0.$$

Û . $h(x) = 1$« @ Î Cauchy z a ª Y \ .

$*$12.Let $f(x)$ and $g(x)$ be continuous functions on $[a,b]$ with $g(x) \geq 0$ for all $x \in [a,b]$. Prove that there is a number ξ in (a,b) such that

$$\int_a^b f(x)g(x)\,dx = f(\xi)\int_a^b g(x)\,dx$$

Û . { r (U ! *the weighted mean-value theorem for integrals*° ¦ (a ª Y \ . $g(x) = 1$« @ Î P ¢ © z a ª Y \ .

§6.Ç P ¢ .

D { B d ¼ ® v ² Y P ¢ = ¢ z ! u ¢ U Y ; À + Ç P ¢ z g ¢ . D ¼ ® Y S Y P ¢ « , ¼ ® r Y 3 P ¢ É ¨ s « , 5 r Y P ¢ z a L p ! s m a L . p (Ü Ú Q © m Ð , CauchyÀ + (Ç P ¢ z g ¢ . ß ß ¼ ® ¿ ! µ ´ N : ¼ ® ~ % Ñ Ö 0 Ì ¨ T, arctan$T$c K) ° P ¢

$$\int_0^T \frac{1}{1+x^2}\,dx$$

Ï ¾ .L C S B 5] p arctan$\infty = \pi/2$. ´) ¼ ® ¨ È Â Ä P ¢

$$\int_0^\infty \frac{1}{1+x^2}\,dx$$

³ ¢ p $\pi/2$.

CauchyÀ + (Ç P ¢ (improper integral)z = ¢ , 0 1 p (: \ C ³ ¢ Q à ¿ % .) 7 ¼ ® Ä) G ´ ¡ . z P ¢ » p ¡ B P ¢ (proper integral). ¼ ® P Y S z Ç P ¢ G

$$\int_a^b f(x)\,dx$$

g ° ,Æ d a, b0 Ï Ì ¨ ° § y $\pm\infty$, µ # É ¨ $f(x)$c ! Y s « .) 7 ¼ ® ° IÏ ¾ y Ç P ¢ , Æ Y S P ¢) 7 ¨ ¼ a Ü :

1)$a = -\infty, b$p Ì ¨ ,# Ñ Ö 0 $T \in (-\infty, b)$, P ¢

$$\int_T^b f(x)\,dx$$

a p ¡ B P ¢ .R (e m

$$\lim_{T\to-\infty}\int_T^b f(x)\,dx$$

U D # p s m ª , y « ¼ ® Ä IY S p µ e m .

2)$b = \infty, a$p Ì ¨ ,# Ñ Ö 0 $T \in (a, \infty)$,P ¢

$$\int_a^T f(x)\,dx$$

a p ¡ B P ¢ .R (e m

$$\lim_{T\to\infty}\int_a^T f(x)\,dx$$

U D # p s m ª , y « ¼ ® Ä IY S p µ e m .