Á %

R Ä 7 $ 7 P ¢ (

1. $\int \cos^7 x \sin x \, dx$.
2. $\int e^x \sin x \, dx$.
3. $\int_1^2 x^3 \ln x \, dx$.
4. $\int_1^2 \tan^{-1} x \, dx$.
5. $\int \cos^{-1} x \, dx$.
6. $\int x^2 \cos x \, dx$.

1 0 k .

? É ¨ u D $[a,b]$ 9 Å s ! e z U Ô « É ¨ ,; f 1 µ U Ô É ¨ ,# f z Y S Î ? V u z ª Î ,1 F p f z ± « É ¨ ,J * ° . ¢ © z U Ì Á Õ $F(u(x))$ Q

$$\frac{d}{dx}F(u(x)) = F'(u(x))u'(x) = f(u(x))u'(x).$$

S µ s

$$\int f(u(x))u'(x)\,dx = F(u(x)) + c = \int f(u)\,du, \qquad u = u(x).$$

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$$\int_a^b f(u(x))u'(x)\,dx = \int_{u(a)}^{u(b)} f(u)\,du.$$

ß ^ : µ } u 0 k z r È ´ N :

´ **1** $\int \sin^3 x \cos x \, dx$.

³ . ? $u(x) = sinx$,J $u'(x) = cosx$.

Î a $=\int (u(x))^3 \cdot u'(x)\,dx = \int u^3\,du = \frac{u^4}{4} + c = \frac{\sin^4 x}{4} + c.$

´ **2** % $\int x\sqrt{a^2 + x^2}\,dx$.

³ . ? $u(x) = a^2 + x^2$,J $u'(x) = 2x$.

Î a $= \frac{1}{2}\int u(x)^{1/2} \cdot u'(x)\,dx = \frac{1}{2}\int u^{1/2}\,du = \frac{1}{2}\cdot\frac{2}{3}\cdot u^{3/2} + c = \frac{1}{3}(a^2+x^2)^{3/2} + c.$

´ **3** $\int \frac{dx}{x(\ln x)^2}$.

³ . 1 $u = \ln x$.J Î a $= -1/u + c = -\frac{1}{\ln x} + c$.

´ **4** % $\int \tan x \, dx$.

³ . ? $u(x) = \cos x$,J

$$\int \tan x \, dx = \int \frac{\sin x}{\cos x} \, dx = -\int \frac{u'(x)}{u(x)} \, dx = -\int \frac{du}{u} = -\ln |u| + c = \ln |\sec x| + c.$$

" ^ ! © À + U ¾ ¨ z ´ N .

´ **5** % $\int_0^1 \frac{du}{(1+u^2)^2}$.

³ . ? $u = \tan x$,$0 \le x \le \pi/2$,J $du/dx = \sec^2 x$.% s

$$\begin{aligned}\int_0^1 \frac{du}{(1+u^2)^2} &= \int_{\arctan 0}^{\arctan 1} \frac{\sec^2 x}{(1+\tan^2 x)^2} \, dx = \int_0^{\pi/4} \frac{dx}{\sec^2 x} \\ &= \frac{1}{2} \int_0^{\pi/4} 1 + \cos 2x \, dx = (\frac{x}{2} + \frac{\sin 2x}{4})|_0^{\frac{\pi}{4}} \\ &= \frac{\pi}{8} + \frac{1}{4}.\end{aligned}$$

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´ **6** % $\int_1^2 \frac{dx}{x\sqrt{1+x}}$.

³ . ? $\sqrt{1+x} = y$,; $x = y^2 - 1$,J $dx/dy = 2y$.Õ 1

$$\begin{aligned}\int \frac{dx}{x\sqrt{1+x}} &= \int \frac{2y}{(y^2-1)\cdot y} \, dy = \int (\frac{1}{y-1} - \frac{1}{y+1}) \, dy \\ &= \ln |\frac{y-1}{y+1}| + c = \ln |\frac{\sqrt{1+x}-1}{\sqrt{1+x}+1}| + c.\end{aligned}$$

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$$\int_1^2 \frac{dx}{x\sqrt{1+x}} = \ln \frac{\sqrt{3}-1}{\sqrt{3}+1} - \ln \frac{\sqrt{2}-1}{\sqrt{2}+1}.$$

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$$\frac{2}{y^2-1} = \frac{1}{y-1} - \frac{1}{y+1}.$$

Q , 1 7 ! B a ¢ ¢ a z ¿ ´ .