Transmission of Probability Theory into China at the end of the Nineteenth Century

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In celebration of the seventieth birthday of Joseph Dauben,
an esteemed scholar and a good friend

Introduction

In spite of the occurrence of many uncertain events in human experience in different civilizations since antiquity, be it in the East or the West, a quantitative approach to probability was not developed until the sixteenth and seventeenth centuries in Western Europe. This peculiar "miss" is particularly notable in the history of Chinese mathematics, even though knowledge and skill in numerical calculation had long been well developed in ancient and medieval China. Probability first came to China in the form of a book, *Jueyi Shuxue* (literally meaning: "mathematics to resolve uncertainty"). It was translated around 1880 through the collaboration of the British missionary John Fryer (1839-1928) and the Chinese mathematician HUA Heng-fang (1833-1902) from an article on probability written by Thomas Galloway (1796-1851) for the *Encyclopaedia Britannica*; this article was published separately as the book *A Treatise on Probability*.

This paper concerns this episode and related issues: it tells the story of how the theory of probability was first introduced into China in the late-nineteenth century. In a way this is not a story of success, but not one of complete failure either. This first book on probability, *Jueyi Shuxue*, introduced a new branch of mathematics to Chinese mathematicians and scientists towards the last years of the old Imperial China, though it seems to have exerted little influence on the subsequent development of probability and statistics which took place in the Chinese Republic. The serious study of probability and statistics, in the modern sense of the subject, only got underway in China in the 1930s, from which point onwards its development was similar to that in the Western world (For an account of the history of statistics in China see for instance [Li, Mo 1993]). Why are we interested in this "unsuccessful" attempt? A main reason is because we look at the episode from the viewpoint of incompatibility of the cultural, social and linguistic contexts. We thus see this "failure" as an instance of a far more general phenomenon, for when a foreign scientific discipline is implanted into an indigenous environment in which the concepts and technical means are foreign and largely incompatible with the scientific language of the time, one should hardly be surprised when this new discipline does not bear fruit immediately.

Many researchers have brought forth scholarly studies of the development of probability theory in the West. Some have analyzed why the subject arose as late as the sixteenth and seventeenth centuries in Europe, even though the notions of

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"probable" or "chance" were by no means foreign to ancient civilizations both in the East and the West, in which games were a familiar part of life since early antiquity. In the case of China, the absence of interest in the theory of probability until the subject was transmitted through *Jueyi Shuxue* in the late-nineteenth century raises a particularly interesting question, especially in view of the many accomplishments of Chinese mathematicians from antiquity to medieval times. We thus turn to address this issue with the benefit of perceptive views offered by other scholars who have been interested in this tantalizing question.

Jueyi Shuxue

The book entitled *Jueyi Shuxue* (決疑數學) [Jueyi Shuxue 1896] has been studied by several historians of mathematics. They have concluded this is a translation based on an article of Thomas Galloway (1796-1851) in the eighth edition of *Encyclopaedia Britannica* of 1859 (see [Bréard 2008], [Guo 1989], [Wang 2006], [Wang, Bréard 2010], [Yan 1990]). Thomas Galloway was a Scottish mathematician who taught at the Royal Military College at Sandhurst beginning in 1823. He later became an actuary with the Amicable Life Assurance Company of London from 1833 to the end of his life. Galloway's text first appeared as an article in the seventh edition of the *Encyclopaedia Britannica* in 1839 and was also published separately as a book [Galloway 1839]. The translation was accomplished through the collaboration of the British missionary John Fryer (傅蘭雅 1839-1928) and the Chinese mathematician HUA Heng-Fang (華蘅芳 1833-1902). This was accomplished in the traditional manner of those days, namely, the Western translator would explain the text in detail to the Chinese translator, who would then study the content as deeply as possible and then write out a Chinese text in consultation with the Western translator. The translation was published in 1896 at the initiation of ZHOU Xue-Xi (周學熙 1866-1947), a well-known Chinese official, politician and industrialist. Although it was already announced in the Chinese magazine *Gezhi Huibian* (格致彙編) that the translation had been finished by the autumn of 1880, it took more than fifteen years before it was finally published. According to what ZHOU's nephew, the mathematician ZHOU Da (周達 1879-1949), said in the preface of a reprinted edition of the book in 1909, only a few copies were printed and their circulation was not very wide (see Section 3) [Guo 1989].

Jueyi Shuxue reproduces the original English text [Galloway 1839] quite faithfully. The content essentially reflects that of a text on probability after the style of Laplace to which the work of Poisson has been added. The introduction begins with the sentence: "The doctrine of probability is an extensive and very important branch of mathematical science, the object of which is to reduce to calculation the reasons which we have for believing or expecting any contingent event, or for assenting to any conclusion which is not necessarily true." After some general discussion of the subject a historical account of the theory from the Pascal-Fermat correspondence to the fundamental treatises of Huygens, Bernoulli and others are given including the landmark classics *Ars Conjectandi* by Jakob Bernoulli (called by the name of James Bernoulli in Galloway's book), followed by the works of Montmort, De Moivre, Bayes, Laplace and Poisson, with the work of De Morgan of 1837 mentioned at the end. By the end of the nineteenth century probability theory had developed well beyond Laplace's work [Schneider 1987], but the translation reflects the enthusiasm the translators still held for Laplace's work.

A conscientious Chinese reader would, through this faithful translation, gain knowledge of the development of probability theory in Europe since the seventeenth century from a comprehensive historical account spanning some ten pages of the book. Still, it is questionable to what extent a Chinese reader in the late-nineteenth century would have been able to comprehend the logical and philosophical context of a subject that was so new and foreign to him. We cite here a representative passage from this historical account:

> The foundations of the mathematical theory of probabilities were laid by Pascal and Fermat about the middle of the 17th century. [...] and a correspondence on the subject took place between these two illustrious geometers, which is preserved in their respective works, and throws some light on the history of mathematics in that age.
>
> About the same period Huygens composed his tract *De Ratiociniis in Ludis Aleæ*, which was first published in the *Exercitationes Geometricæ* of Schooten in 1658. This was the first systematic treatise which appeared on the doctrine of chances. It contained an analysis of the various questions which had been solved by Pascal and Fermat, and at the end five new questions were proposed, the solutions of which, simple as they may now appear, were then attended with considerable difficulty. The analysis of two of them was in fact given for the first time by Montmort, half a century after their publication. [...]
>
> James Bernoulli appears to have been the first who perceived that the theory of probability may be applied to much more important purposes than to regulate the stakes and expectations of gamesters, and that the phenomena, both of the moral and physical world, anomalous and irregular as they appear when viewed in detail, exhibit, when considered in large numbers, a constancy of succession which renders their occurrence capable of being submitted to numerical estimation. The *Ars Conjectandi*, published in 1713, seven years after the death of the author, contains a number of interesting questions relative to combinations and infinite series; but the most remarkable result which it contains is a theorem respecting the indefinite repetition of events, which may be said to form the basis of all the higher applications of the theory. It consists in this, that if a series of trials be instituted respecting an event which must either happen or fail in each trial, the probability becomes greater and greater, as the number of trials is increased, that the ratio of the number of times it happens, to the whole

number of trials, will be equal to its *a priori* probability in a single trial; and that the number of trials may be made so great as to give a probability, approaching as nearly to certainty as we please, that the difference between the ratio of its occurrences to the number of trials, and the fraction which measures its *a priori* probability, will be less than any assigned quantity. Bernoulli informs us, that the solution of this important theorem had engaged his attention during a period of twenty years.

In the interval between the death of Bernoulli and the appearance of the *Ars Conjectandi*, Montmort published his *Essai d'Analyse sur les Jeux de Hazard*. The first edition was in 1708; the second, which is considerably extended, and enriched by several letters of John and Nicolas Bernoulli, appeared in 1713. [...]

About the same time, Demoivre began to turn his attention to the subject of probability, and his labours, which were continued during a long life, contributed greatly to the advancement of the general theory, as well as the extension of some of its most interesting applications. [...]

The objects and important applications of the theory of probabilities having been made known by the works now mentioned, the subject has ever since been regarded as one of the most curious and interesting branches of mathematical speculation, and accordingly has received more or less attention from almost every mathematician of eminence. A great variety of questions connected with it and especially relating to lotteries, are interspersed in the volumes of the *Paris* and *Berlin Memoirs*, (particularly the latter,) by John and Nicolas Bernoulli, Euler, Lambert, Beguelin, and others. D'Alembert has likewise treated of the theory in several of the volumes of his *Opuscula*; and it is not a little remarkable, that in some instances its first principles should have been misunderstood by so ingenious and profound a writer. [...]

[...] The *Transactions of the Royal Society* for the years 1763 and 1764, contain two papers by the Rev. Mr. Bayes, with additions to the latter by Dr. Price, which deserve to be noticed, inasmuch as the principles on which the probability of an event is determined, when the event depends on causes of which the existence and influence are only presumed from experience, are there for the first time correctly laid down. The question proposed and solved by Bayes was this: a series of experiments having been made relative to an event, to determine the presumption there is, that the fraction which measures its probability falls within given limits.

One of the earliest applications of the theory of probability was to determine, from observations of mortality, the average duration of human life, and the value of pecuniary interests depending on its continuance or failure. [...]

The application of the theory of probability to the subject of ju-

> risprudence, and the verdicts of juries and decisions of tribunals, has been discussed by the Marquis Condorcet in various articles in the *Encylopédie Méthodique*; [...]
>
> The analysis of probability has been applied with signal advantage in many researches of Natural Philosophy, but especially in appreciating the mean errors of observations. [...] The mean errors of observations were treated as questions of probability by Lagrange in the Turin *Memoirs* for 1773; but it is to Laplace that the theory owes its principal extension and most important results. [...]
>
> Laplace's great work, the *Théorie Analytique des Probabilités* , first published in 1812, is one of the most remarkable productions that has ever appeared in abstract science. The principles of the calculus, as well as the peculiar methods of analysis which it requires, and the most interesting and difficult questions which it presents, are here discussed in a far more general manner than had been attempted by any former writer on the subject; and it may be said, accordingly, to have placed the theory under an entirely new aspect. [...]
>
> Next to the *Théorie Analytique* of Laplace, the most important work which hitherto appeared on the subject of probability is the recent one of Poisson, entitled, *Recherches sur la Probabilité des Judgements*, (Paris 1837.) [...]
>
> It is in these two works of Laplace and Poisson that the higher and more abstruse parts of the theory of probabilities must be studied. A very clear exposition of the principles, accompanied with many interesting remarks on the uses and applications of the theory, is given by Lacroix in his valuable little work, *Traité Elémentaire du Calcul des Probabilités*, Paris 1822.
>
> Since the time of Demoivre, the English treatises on the general theory of probability have neither been numerous, nor, with one or two exceptions, very important. [...] but by far the most valuable work in the language is the Treatise in the *Encyclopedia Metropolitana*, by Professor De Morgan, 1837. In this very able production, Mr. De Morgan has treated the subject in its utmost generality, and embodied, with a moderate compass, the substance of the great work of Laplace.

It is interesting to note a passage in the introduction that has this to say about probability and gambling:

> In fact, most of the questions of this class to which the calculus can be applied, are connected with lotteries and games of hazard. The results obtained from the analysis of such questions cannot be considered as being of any great value in themselves, but they frequently throw light on subjects of far higher importance which present analogous combinations. It is true that the mathematical theory comes in aid of moral considerations, and demonstrates the ruinous tendency of gambling even

> when the conditions of the play are equal, mathematically speaking; but, unfortunately, those who indulge a passion for this vice are seldom capable of appreciating the force of such arguments.

This expression of a negative attitude towards gambling perhaps had an appeal for the Chinese translator, who was brought up in a Confucian tradition in which gambling was censured by public opinion as a vice that led to corruption of moral standards.

Historical Background of the Translation

The translation of Western books on science and mathematics went on as a fervent activity in Imperial China in the latter half of the nineteenth century. One has to look at it in the historical context of what the country went through during that period, which the historian Immanuel HSÜ (徐中約 1923-2005) describes as a "search for a way to survival in the new world that had been forcibly thrust upon China by the West after the mid-19th century" [Hsü 1970/1995]. The Chinese, "burdened by tradition and heritage, and as yet ignorant of the nature of the Western world, groped in the dark." HSÜ maintains that "the dynamics of change suggests that modern Chinese history is not characterized by a passive response to the West, but by an active struggle of the Chinese to meet the foreign and domestic challenges in an effort to regenerate and transform their country from an outdated Confucian universal empire to a modern national state, with a rightful place in the family of nations" [Hsü 1970/1995].

To see this in a wider historical context one has to look at the three waves of transmission of Western science into China. The translation of Euclid's *Elements* by XU Guang-qi and Matteo Ricci in 1607 led the way during the first wave [Siu 2011]. As pointed out by Siu [Siu 1995/1996]: "The gain of this first wave seemed momentary and passed with the downfall of the Ming Dynasty. Looking back we can see its long-term influence, but at the time this small window which opened onto an amazing outside world was soon closed again, only to be forced open as a wider door two hundred years later by Western gunboats that inflicted upon the ancient nation a century of exploitation and humiliation, thus generating an urgency to know more about the Western world."

The second wave came in the wake of the first wave and lasted from the mid-seventeenth century to the mid-eighteenth century. Instead of Chinese scholar-officials the chief promoter was Emperor Kangxi of the Qing Dynasty (reigned 1662-1722). Instead of Italian and Portuguese Jesuits the Western partners were mainly French Jesuits, the so-called "King's Mathematicians" sent by Louis XIV, the "Sun King" of France (reigned 1643-1715), in 1685 [Du, Han 1992]. Emperor Kangxi learnt assiduously Western mathematics and astronomy from these French Jesuits. Later in 1713 a group of selected Chinese officials and scholars was summoned to form the Office of Mathematics that was established in *Mengyangzhai* (Studio for the Cultivation of the Youth 蒙養齋), a place situated in the garden

Changchunyuan (Garden of the Exuberant Spring 暢春園) inside the Imperial Palace. Besides serving as a school for taking lessons in mathematics, astronomy and science, the main task of this Office was to compile the monumental treatise *Lüli Yuanyuan* (Origin of Mathematical Harmonics and Astronomy 律曆淵源), comprising three parts: *Lixiang Kaocheng* (Compendium of Observational Computational Astronomy 曆象考成), *Shuli Jingyun* (Collected Basic Principles of Mathematics 數理精蘊), and *Lülü Zhengyi* (Exact Meaning of Pitchpipes 律呂正義). The treatise was completed in 1722/1723. The second book *Shuli Jingyun* includes both traditional Chinese mathematics, the part that was still extant and was understood at the time, as well as Western mathematics, most likely from the "lecture notes" prepared by the missionaries for Emperor Kangxi during his ardent study of mathematics in the 1680's. Interested readers will find a more in-depth discussion of this second wave in a paper of Catherine Jami [Jami 2002], as well as in [Jami 2012]

The third wave came in the last forty years of the nineteenth century in the form of the so-called "Self-strengthening Movement" after the country suffered from foreign exploitation during the First Opium War (1839-1842) and the Second Opium War (1856-1860). This time the initiators were some officials of high rank led by Prince Gong (Yixin 恭親王奕訢 1833-1898) with contributions from Chinese scholars and Protestant missionaries coming from England or America [Chan, Siu 2012, Siu 2009, Xiong 1994].

The theme and mood of the three waves of transmission of European science into China were reflected in the respective slogans prevalent in each period. In the first part of the seventeenth century the idea was: "In order to surpass we must try to understand and to synthesize (欲求超勝必須會通)." In the first part of the eighteenth century it became: "Western learning has its root in Chinese Learning (西學中源)." In the latter part of the nineteenth century the slogan took on a very different tone: "Learn the strong techniques of the '[Western] barbarians' in order to control them (師夷長技以制夷)." In a paper on European science in China, Catherine Jami says: "[...] the cross-cultural transmission of scientific learning cannot be read in a single way, as the transmission of immutable objects between two monolithic cultural entities. Quite the contrary: the stakes in this transmission, and the continuous reshaping of what was transmitted, can be brought to light only by situating the actors within the society in which they lived, by retrieving their motivations, strategies, and rationales within this context." [Jami 1999].

Headed by Prince Gong and supported strongly by some officials of high rank, schools were established to teach Western science and mathematics, and offices were set up to translate Western texts in science and mathematics. In particular, an establishment known by the name of *Tong Wen Guan* (College of Foreign Languages 同文館) was set up in 1862 by decree, with the section on mathematics and astronomy established in 1866. The establishment of *Tong Wen Guan* was at first intended as a language school to train interpreters but later developed into a college of Western learning, along with other colleges of similar nature that sprouted in other cities like Shanghai, Guangzhou, Fuzhou, Tianjin, along with the establishment of arsenals, shipyards and naval schools.

At the beginning of the nineteenth century a new wave of Christian missionaries came to China, starting with Robert Morrison (1782-1834) of the London Missionary Society of London in 1807. As pointed out by Benjamin Elman [Elman 2005]: "Although their enthusiasm for China was often fired by evangelicalism back home, British and American Protestants in China swiftly recognized that Chinese literati were interested in the sciences that the missionaries accepted as part of their Christian heritage. Like their Jesuits predecessors, British and American missionaries viewed science as emblematic of their superior knowledge systems."

Once the missionaries had this view they expanded their non-evangelical activities to include education, medical work and publication. In 1868 the English missionary John Fryer was recruited to work in the translation department of the Jiangnan Arsenal (江南製造局) in Shanghai. Fryer was the most prolific translator who took part in completing more than a third of the total output in this department. During the period 1868-1896, he collaborated with the Chinese mathematician HUA Heng-Fang, who was his colleague at the Jiangnan Arsenal. (See [Bennett 1967] and [Horng 1993] for a detailed account of Fryer and HUA respectively.) The translation of *Jueyi Shuxue* was accomplished during this period in which the Chinese government, as well as many Chinese scholars, harboured a fervent and urgent desire to learn from the Westerners in order to resist foreign exploitation of their country.

After *Jueyi Shuxue* was published in 1896, Fryer issued it again in 1897 through his *Gezhi Shushi* (Chinese Scientific Book Depot, literally: bookshop for investigating things and extending knowledge 格致書室), which he set up in 1884 as a retail outlet of the Jiangnan Arsenal [Elman 2005]. The book was reprinted in 1909 by the Chinese mathematician ZHOU Da, who had great interest in the book, a copy of which he inherited from his uncle who first printed it (see Section 2). Still, ZHOU Da lamented that it was not given adequate attention so that circulation was not widespread. Despite this admirable effort of ZHOU Da, who stressed in a preface to the reprinting how important this theory was to mankind, the translated book did not seem to capture much attention at the time. The fact that the book remained unpublished for fifteen years after it was translated may reflect doubts of the translators themselves about its suitability for the indigenous scientific community. Or, it may possibly reflect their lack of confidence in having a strong enough grasp of this new branch of mathematics. This may explain why the book does not present an introduction written by Fryer nor by HUA, whereas the two other well-known and popular translated books by them contained such explanatory introductions. This famous pair of collaborators produced *Daishu Shu* (Method of Algebra 代數術) in 1873 and *Weiji Suyuan* (The Origins of the Differential and Integral Calculus 微積溯源) in 1874. These were, respectively, translations of the articles on "Algebra" and "Fluxions" in the eighth edition of the *Encyclopaedia Britannica*, which were written by the Scottish mathematician William Wallace (1768-1843). Both texts were influential in transmitting these new subjects, algebra and calculus, into China at the time (see [Hu 1998] for more on the translation of those two books). In contrast, the new subject of probability theory was not met with similar enthusiasm from the scientific community, partly it seems because the translators themselves had doubts about how to synthesize the subject with the Chinese mathematical tradition!

Reception of the Translation at the Time

In 1896 a leading Chinese intellectual from this period, LIANG Qi-Chao (梁啟超 1873-1929), compiled a list of Western books for recommended reading. He published this in the periodical *Shiwu Bao* (The China Progress 時務報), which he had helped to establish in that same year. *Jueyi Shuxe* was included on this list, even though the translation was not yet published, as LIANG Qi-Chao pointed out: "The translation of *Jueyi Shushu* (決疑數術 which should be *Jueyi Shuxue*) was completed but the book is not yet published [by the Translation Bureau of the Jiangnan Arsenal]. The Westerners apply this method to insurance and other matters. There was once an article published in *Gezhi Huibian* (格致彙編) to introduce the subject. The article is brief and cannot give the details." This magazine is the 1891 Spring issue of *Gezhi Huibian* [literally: compendium for investigating things and extending knowledge] with the English title *The Chinese Scientific and Industrial Magazine* [Gezhi Huibian], a Chinese magazine published from 1876 to 1892 by the English missionary John Fryer (1839-1928) and the Chinese scholar XU Shou (徐壽 1818-1884) in Shanghai, and was modelled after an earlier publication of a similar nature, the *Peking Magazine*.

In the latter part of the nineteenth century some Western missionaries published periodicals to propagate Western learning in old imperial China as a means of spreading Christian faith. Thus, the periodical *Zhongxi Wenjian Lu* (Record of News in China and the West 中西聞見錄) with the English title *Peking Magazine* was founded in August of 1872 by the American missionary William Alexander Parsons Martin (1827-1916) and the English missionary Joseph Edkins (1823-1905), supported by the Society for the Diffusion of Useful Knowledge in Beijing [Peking]. At the back of the title-page of the first issue the objective in publishing this periodical was clearly stated in a short note, further elaborated in the ensuing preface. The short note says, "*Zhongxi Wenjian Lu* [*Peking Magazine*] adopts the practice and format of newspapers in the Western world in publishing international news and recent happenings in different countries, as well as essays on astronomy, geography and *gewu* [science, literally meaning " investigating things"]. The magazine will be published once every month. Any gentleman, Chinese or Westerner who gathers new information or has his own views to express, is invited to submit it to the editors at the *Shi Yiyuan* (Charity Hospital) of *Mishi* [*Street*] (Rice Market Street). The editors will select those items that are considered to be fit for print. In this way, new information will be attained through collective effort to benefit more people so as to enable them to become more and more knowledgeable" [Zhongxi Wenjian Lu] .Upon the closing down of the Society for the Diffusion of Useful Knowledge in August of 1875, the periodical was also terminated after thirty-six issues, to be revived in 1876 in the form of another periodical, *Gezhi Huibian*.

The aforementioned article in the 1891 Spring issue of *Gezhi Huibian* titled “Using mathematics to resolve uncertainty” was probably the only other mentioning of the subject besides *Jueyi Shuxue* at the time. No author’s name is attached to the article. It was likely to have come from the pen of Fryer himself. Twenty-six years later in 1917 another article on probability appeared in the journal *Kexue* (Science 科學), which was the official publication of a newly established organization *Zhongguo Kexue She* (Science Society of China 中國科學社) founded in 1914 by a group of Chinese students studying in the United States who were devoted to bringing about progress and development of their motherland and who shared the view that science would be the key to move the country forward. The journal *Kexue* was established with the intention of promoting science among the Chinese people, to “promote science, encourage industry, authorize terminologies, and spread knowledge” [Buck 1980]. The author of this article on probability is HU Minfu [HU Ming-fu] (胡明復), who was the first Chinese to earn a doctorate in mathematics abroad (from Harvard in 1917) as well as the first Chinese mathematician to publish a paper in an international journal of fame [Linear integro-differential equation with a boundary condition, *Trans. Amer. Math, Soc.*, 19(4) (1918), 1-43.] HU Ming-fu returned to China after 1917 and made great contributions to the scientific development of the country before he met a tragic untimely death while swimming in Wuxi in 1927 [Zhang 1991]. Going further than the 1891 article on probability in *Gezhi Huibian*, HU’s article on probability not only explained what the subject was about but also touched upon philosophical aspects and the role it plays in scientific thinking [Hu 1917]. Actually, the article was published as the third part under the general heading of methodology in science.

In a different direction, LIANG Qi-Chao also applied his knowledge of probability to another area of human activity. In one of his political essays, entitled “Nü xue (Women’s schooling 女學)” and published in the March issue of *Shiwu Bao* in 1897, he again referred to the book *Jueyi Shuxe*:

> *Mengzi* (孟子) says, “To dwell in idleness without instruction is a life akin to that of the birds and beasts.” [...] In general, the people in a state are bound to let each one have an occupation. If everyone can support himself, then the state can be governed extensively. If there are those who cannot do this, then its strength and weakness varies with the number of people who have no occupation. What is the reason for this? Those persons who have no occupation, must be supported by those who have an occupation. If they are not supported, then those who have no occupation are in a perilous situation, if they are supported, then those who have an occupation are in danger. This is what it means. What the Westerners in their translations called ‘production and distribution’ is what is expressed in the *Great Learning* (大學) with

> "Those who produce are numerous, those who eat are few." *Guanzi* (管子) said: "If a single peasant is not engaged in farming, someone will suffer hunger; if a single woman does not engage in weaving, someone will suffer cold." These are not empty words. The above is like taking the entire population of a state and its material production, and by the *Mathematical Art of Probability* (*jueyi shushu* 決疑數術), increasing and decreasing, dispersing and adding it. The obtained ratio is indeed like this: in China, those who sponge on others are half of those who create profit. [...] There are two hundred million women who belong entirely to the consumers and of whom not a single one is productive. Because they cannot support themselves and have to be supported by others, men elevate them like dogs, horses, or slaves, which makes women's life even more miserable." (This English translation is taken from [Bréard 2008], with slight modifications; the full essay is in [Liang 1936].)

Even though LIANG Qi-Chao referred to the notion of probability in his political essay on reform in advocating literacy for women and hence in defending women's rights, it seems that he did not entirely grasp the concept of probability and regarded probability just as simply a proportion! This kind of superficial understanding of probability by a well-educated scholar of the time is not surprising. The actual influence of the book *Jueyi Shuxue* seems to have been rather minimal, and subsequent studies and works on probability and statistics suggest little direct impact from the book. It turns out that most of the terms the translators coined in probability theory did not survive so that Chinese terminologies adopted in later textbooks were different. An obvious example is "*jueyi shuxue*" (決疑數學), which was later replaced by "*gailü lun*" (概率論) after the temporary adoption of various other translated terms.

Lack of Influence of the Translation

A major obstacle for Chinese translations of modern mathematics texts in the nineteenth century was the lack of existing terminology. This meant not only that new Chinese terms had to be coined but also that Western expressions had to be adopted in a mathematical symbolism that was foreign to Chinese readers. Thus, one reason for the lack of influence of *Jueyi Shuxue* might have to do with the mathematical language adopted in the translation. This, after all, was not a complete symbolic format already commonly adopted in Western mathematical literature but rather a hybrid between the Chinese format adopted in traditional Chinese mathematical texts and some newly coined symbols with a Chinese flavour. To our modern eyes this would indeed look awkward and impede facility in computation or even ease in comprehension. Let us look at some excerpts from the *Jueyi Shuxue*:

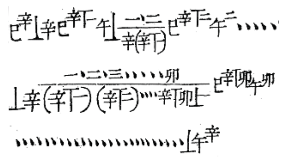
(1) Section II, Article 13: Application of the binomial theorem.

This deals with the expansion of the expression $(p+q)$ to the power h. The original text says:

In order to place the proposition now demonstrated in a clearer light, let us consider separately the different terms of the development of $(p+q)^h$, namely,

$$\begin{aligned} p^h &+ hp^{h-1}q + \frac{h(h-1)}{1\cdot 2}p^{h-2}q^2 + \cdots \\ &+ \frac{h(h-1)(h-2)\cdots(h-n+1)}{1\cdot 2\cdot 3\cdots n}p^{h-n}q^n \\ &+ \cdots + q^h. \end{aligned}$$

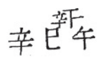
The Chinese translation rendered the formula in the following form:



The original text goes on to say:

The first term p^h expresses the probability that the event E will occur in every one of the h trials. The second term $hp^{h-1}q$ expresses the probability that E will occur $h-1$ times, and F once, without distinction of order; that is to say F may happen at the first or last or any intermediate trial.

The Chinese translation rendered the second term in the following form:



(2) Section VIII, Article 96: Method of computing the value of the integral which expresses the probability.

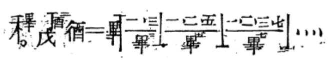
This deals with the computation of the famous integral of e to the power minus x squared. The original text says:

The integral $\int e^{-t^2}dt$ is computed as follows. Developing the exponential e^{-t^2} in a series of the ascending powers of t^2, and integrating the successive terms between the limits $t=0$ and $t=\tau$, we find

$$\int_0^\tau e^{-t^2}dt = \tau - \frac{\tau^3}{1\cdot 3} + \frac{\tau^5}{1\cdot 2\cdot 5} + \frac{\tau^7}{1\cdot 2\cdot 3\cdot 7} + \&c.\,;$$

a series which converges rapidly when τ is less than unity.

The Chinese translation rendered the integral in the following form:



This kind of Chinese-Western mixture of symbolic language was adopted in all the translated mathematical texts at the time. For a student of mathematics today this hybrid certainly looks awkward, however, with sufficient practice familiarity can still be attained. Still, Chinese mathematicians in the nineteenth century might not have regarded this hybrid style as a crucial difficulty in learning the mathematics. Thus, it seems that hindrance to the development of probability theory in ancient and medieval China arose more for cultural and social reasons, to which we now turn.

As HUA Heng-Fang might have anticipated, the concept of probability was not readily received by the indigenous scientific community of his time. With algebra there was the tradition of solving equations in ancient and medieval China, in fact at a level that was quite advanced when compared with that of the Western world in the corresponding period. Similarly, with calculus there was the notion of infinitesimals already used in the computation of areas and volumes as explained by LIU Hui (劉徽) in the third century. But the quantitative notion of uncertainty was totally absent in all Chinese mathematical classics up to that point.

This peculiar "miss" in the history of Chinese mathematics presents a baffling question in two respects. The first concerns an "internal" issue, since the needed mathematical knowledge and skill in numerical calculation had long been developed in ancient and medieval China (see Section 6). The second aspect reflects an "external" issue, since probability theory has so many applications. It is commonly agreed that traditional Chinese mathematics paid much attention to demands of solving real world problems. Indeed, this exemplified a basic tenet of traditional Chinese philosophy of life shared by the class of *shi* (intellectuals 士), namely, self-improvement and social interaction (經世致用). Had the Chinese mathematicians become aware of the fundamental significance and applicability of the notion of probability they would doubtless have devoted their efforts to its investigation. One might, of course, argue that the actual "real-life" problems in China of the nineteenth century were not yet developed to the stage that gave rise to probability theory in the European world of the seventeenth and eighteenth centuries.

The Notion of Probability in Ancient and Medieval China

Perhaps this question of the peculiar "miss" has to be viewed not just within the Chinese context alone but also against the full background of developments in the West as well.[1] As noted at the outset of this essay, despite the role of uncertain events throughout human history, a quantitative approach to probability was not developed until the sixteenth and seventeenth centuries in Western Europe. For instance, gambling as a human activity has a very long history, an early example of risk taking. According to Florence Nightingale David "the real problem which

[1]For relevant studies of this, see [Bernstein 1996], [Daston 1988], [David 1962], [Hacking 1975/2006], [Hald 1990], [Hald 1998], [Maistrov 1974], [Meusnier 1996], [Plato 1994], [Sheynin 1974], [Stigler 1986].

confronts the historian of the calculus of probabilities is its extremely tardy conceptual growth– in fact one might almost say, its late birth as an offspring of the mathematical sciences” [David 1962].

Mark Elvin points out [Elvin 2002/2010, Elvin 2004]–following Alistair Crombie’s six styles of scientific thinking in the European world (namely, “postulational,” ‘experimental’, “hypothetical modelling,” “taxonomy,” “probabilistic,” and “historical derivation”) [Crombie 1994]–that probabilistic and statistical thinking was the last that came to maturity. One might for this reason regard this style as perhaps the least natural kind of scientific thinking, thus an approach that needs time to incubate, be it in the West or the East. In his account of the prehistory of the theory of probability, Oscar Sheynin mentions a saying of Aristotle: “Evidently, none of the traditional sciences busies itself about the accidental, [. . .] this (the accidental) none of the recognized sciences considers, but only sophistic [. . .].” Sheynin points out that Aristotle was the first to attempt an explanation of chance [Sheynin 1974]. In a paper that discusses the possible and the probable [Sambursky 1956], Shmuel Sambursky points out an interesting feature of Greek philosophy in general and its mathematics in particular that caused their “miss” in understanding the intriguing and important concept of probability, namely, the cosmic outlook of the ancient Greeks which “saw precision restricted to the heavens and failed to see laws in the fluctuations and irregularities on earth.” [Sambursky 1956] A breakdown of this barrier between “heavenly” and “terrestrial” phenomena in the sixteenth/seventeenth centuries freed the human mind so that the investigation of chance events was on the agenda. The ancient Greeks perceived mathematics as an idealistic world of “mathematical reality” for pure intellectual pursuit and presented mathematics as a beautiful and perfect form that embodied truth with certainty. In Plato’s dialogue *Theaetetus* [Plato 1921] Socrates said, “If Theodorus, or any other geometrician, should base his geometry on probability, he would be of no account at all.” Drawing definite conclusions via deductive logic in Greek mathematics is to be contrasted with what Plato defined in another dialogue, *Phaedrus* [Plato 1914], as “accepted by the people because of its likeness to truth”, which reminds one of the German term for probability —“Wahrscheinlichkeit”.

In the Chinese scene there were likewise instances in philosophy, in mathematics and in social practices that could have hinted at probabilistic thinking, but this had not germinated and bloomed into a mathematical theory as it did in the West. No recognizable mathematical account on the notion of probability appeared in the mathematical literature in China before its transmission from the West through *Jueyi Shuxue*. Mathematically speaking it is known that Chinese mathematicians developed much earlier than Pascal what is commonly known as Pascal’s triangle, solved problems related to permutations and combinations, and for administrative reasons treated problems of a statistical nature since early times.

Chinese mathematicians in the Song Dynasty and the Yuan Dynasty developed the arithmetical triangle, known in the Western world as Pascal’s triangle (but actually the triangle was not first discovered by Pascal either). They used this initially as a means for extracting square and cube roots, and they later applied it to solving polynomial equations, working with finite differences and summing finite arithmetical series. The triangle can be dated back to the beginning of the eleventh century in the work of JIA Xian (賈憲 early 11th century), who employed it in the extraction of square and cube roots. It is fortunate that the work of JIA Xian, which was lost, was reported and appeared first in print in the mathematical treatise *Xiangjie Jiuzhang Suanfa* (Detailed Explanations of the Nine Chapters on the Mathematical Method 詳解九章算法) of YANG Hui (楊輝 1238-1298) in 1261, and the triangle of binormial coefficients appeared again in another mathematical treatise *Siyuan Yujian* (Jade Mirror of Four Elements 四元玉鑑) of ZHU Shi-Jie (朱世傑 1249-1314) in 1303, who applied it to polynomial equations, finite differences and finite series [Li 1999, Shen 2000].

Folklore wisdom depicted in some anecdotes embraced an awareness of chance events. In the old book *Thirty-six Stratagems* (三十六計) the Stratagem Number 27 known as "Faked madness but not insane (假痴不癲)" [Ma, Tong 2007] tells the anecdote of how General DI Qing (狄青 1008-1057) of the Northern Song Dynasty in the 11th century boosted the morale of his army before the battle against NONG Zhi-Gao (儂智高 Nungz Ciqgau 1025-1055), the chieftain of the ethnic group of *Zhuang* (壯族) in the Southwestern part of China, by tossing 100 coins all with heads up! The whole army hailed this omen of success and marched into battle with full confidence. General DI ordered that the 100 coins be nailed to the ground and covered up by a silk cloth to await a triumphant return. The army did return in triumph, only to find that all the one hundred specially prepared coins had heads on both sides! This anecdote indicates that people in the old days knew that the chances for certain events were slim.

In Book 18 of the *Mengxi Bitan* (Dream Pool Essays 夢溪筆談), compiled by SHEN Kuo (沈括 1031-1095) in 1088, there is recorded the following problem in counting the number of possible situations in chess:[2]

> The story-tellers say that I-Hsing (一行) once calculated the total number of possible situations (局) in chess, and found he could exhaust them all. I thought about this a good deal, and came to the conclusion that it is quite easy. But the numbers involved cannot be expressed in the commonly used terms for numbers. I will only briefly mention the large numbers which have to be used. With two rows and four pieces the number of probable situations will be 81 different kinds. With three rows and nine pieces the number will be 19,683. Using four rows and sixteen pieces the number will be 43,046,721. For five rows and twenty-five pieces, the number will be 847,288,609,443 $\cdots$. Above seven rows we do not have any names for the large numbers involved. When the whole 361 places are used, the number will come to some figures (of the

[2]This English translation is taken from [Needham 1959/1970].

order of) ten thousand written fifty-two times ($10,000^{52}$ 連書萬字五十二).

One problem in *Shushu Jiuzhang* (Mathematical Treatise in Nine Sections 數書九章) of QIN Jiu-Shao (秦九韶 c.1202-1261) in 1247 made use of the idea of sampling in collecting grains for taxation [Xu, Zhang 1995]:

> When a peasant paid tax to the government granary in the form of 1534 *dan* (石) of rice, it was found out on examination that a certain amount of rice with husks was present. A sample of 254 grains was taken for further examination. Of these 28 grains were with husks. How many genuine grains of rice were there, given that one *shao* (勺) contains 300 grains?" (Note: In the mensuration system of the Song Dynasty, 1 *dan* = 10 *dou* (斗) = 100 *sheng* (升) = 1000 *ge* (合) = 10,000 *shao*. According to tradition recorded in *Jiuzhang Suanshu* (Nine Chapters on the Mathematical Art 九章算術), a grain of rice with husk was counted as half a grain of rice.) [The answer is given as: 4,348,346,456 grains, out of the original 1534 x 10,000 x 300 = 4,602,000,000 grains.]

In the late-seventeenth century CHEN Hou-Yao (陳厚耀 1660-1722) treated in his *Cuozong Fayi* (The Meaning of Methods for Alternation and Combination 錯綜法義) [Chen late 17^{th} century] problems of permutations and combinations in connection with divination (trigrams, hexagrams), the ten heavenly stems (天干) and the twelve earthly branches (地支) to form the astronomical sexagesimal cycles, as well as with dice throwing and card games. (See [Bréard 2012] for a more detailed discussion.) In the foreword to his treatise, CHEN Hou-Yao pointed out that even though all necessary mathematical techniques had already been provided by the famous ancient mathematical treatise *Jiuzhang Suanshu* (believed to be compiled between the second century B.C.E. and the second century), there was no discussion in the treatise of any method to deal with "alternation and combination." The author was among the group of scholars summoned by Emperor Kangxi to work on the compilation of *Lüli Yuanyuan*; in fact, it was CHEN himself who proposed to Emperor Kangxi that such a treatise ought to be compiled. This book is perhaps the first systematic account of combinations and permutations in China, and its contents are very original. However, it is not known whether the author conceived of it by himself, or whether the material was already known at the time in China, or whether the author might have learnt the subject from Emperor Kangxi, who in turn might have learnt it from the French Jesuits during the second wave of transmission of Western science into China. Judged by the fact that the content was not included in the mathematical treatise *Shuli Jingyun*, the latter circumstance seems rather unlikely. That makes the question even more intriguing as to whether CHEN Hou-Yao came upon this subject by himself.

The Status of the Notion of Probability in China

Games of chance were never unfamiliar to the Chinese, even though gambling practice was officially banned by different rulers throughout the long history of China [Guo, Xiao 1995]. This practice of an official ban might explain the lack of interest of the scholarly class which did not deign to investigate and discuss problems relating to chance as a scholarly pursuit. In the Song Dynasty a practice known by the name of *guan pu* or *pu mai* (gambling-based sale 關撲, 撲賣) [Luo, Xu 1994] was not uncommon to boost the sale of goods by shop-owners. If a customer won a coin-tossing game, then he got back his stake together with the goods for free, while if he lost the game then he lost his money but got no goods. It was apparent that those shop-owners had a good notion of the odds (even if they might not know how to calculate the odds theoretically) and the strategy in playing the games in order to stay in business, but that they kept the knowledge to themselves for their own advantage.

Divination (占卜) and fortunetelling may lead to probabilistic thinking but again such knowledge might be covered up in secrecy by those who practiced it as a profession. (On the other hand, divination might also lead to the opposite by thinking that human affairs are destined by the deities so that it is meaningless to investigate the notions of chance and risk!) Physiognomy (面相學), which had long been practiced in China, can be regarded as related to some kind of statistical thinking. The idea of insurance, and thence the related idea of quantification of probabilistic occurences, might have arisen in businesses, for example, in *yin hao* (銀號) or *qian zhuang* (錢莊) or *piao hao* (票號) (Chinese versions of private banks). Another possible business sector in medieval China might have been *biao ju* (鏢局) or *biao hang* (鏢行) (Chinese versions of courier services with military escort). Unfortunately, detailed documentary evidence of such transactions is not easy to come by.

In his very thorough analysis of the situation of probabilistic thinking in premodern China, Mark Elvin [Elvin 2002/2010] points out that there may have been more than what is now known from writings. As a kind of "internal" obstacle he points out that "the cosmos was seen by not a few as an immense system of numbers, and there was a general preoccupation with chance and hazard; but even in the thought of Wang Chong (王充 27-97), who seems to have been the most lucid of the philosophically obsessed in this domain, the notion of an inherent character of luckiness or unluckiness in individuals blocked off easy access to the idea of objective equiprobability as between different players" [Elvin 2002/2010]. In another detailed and comprehensive study on probability and statistics in China, Andrea Bréard suggests that the *Jueyi Shuxue* was "a linguistic and epistemological failure in attempting to translate unfamiliar phenomena into an existing taxonomic system" [Bréard 2008]. Because the classical doctrines of probability theory were transmitted into China "as a body of knowledge without the same time-sequence they had in the West," Chinese readers could not comprehend the theory in its logical and philosophical context. Andrea Bréard also refers to the discussion by

Mark Elvin and concluded that further scrutiny is needed [Bréard 2008].

Having mentioned all these factors we have to admit that in the history of Chinese mathematics the kind of probing into mathematical concepts, purely for the curiosity of intellectual pursuit, was not as marked as in the West. The axiomatic approach to the study of geometry is one example; the investigation of the solvability by radicals of polynomial equations instead of just obtaining highly accurate methods of solving them is another; and (probably!) the study of probability starting from games of chance is a third. It is rather ironic to find that an essential feature of Greek mathematics, which caused the "miss" of a development of probabilistic thinking in the ancient world (as described in Section 6), ultimately evolved in Europe and blossomed into an intellectual pursuit for curiosity's sake as well as for the "mathematization" of the physical world until in the sixteenth/seventeenth centuries probability theory developed into an important branch of mathematics. The absence of such a feature in Chinese mathematics during ancient and medieval times turned out to be, oddly enough, one of the reasons for this "miss." In any event, the development of probabilistic thinking failed to take root within the Chinese setting, so that the subject had to wait a long time before it was first transmitted from the West toward the end of the nineteenth century!

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